**Chapter 10.1: Comparing Two Proportions**

*The sampling distribution of a difference between two proportions*

Why might we want to compare two proportions?

We start with two different samples:

|  |  |  |  |
| --- | --- | --- | --- |
| **Population or treatment** | **Parameter** | **Statistic** | **Sample size** |
| 1 |  |  |  |
| 2 |  |  |  |

The sampling distribution of $\hat{p}\_{1}-\hat{p}\_{2}$

|  |
| --- |
| Choose an SRS of size $n\_{1}$ from population 1 with a proportion of successes $p\_{1}$ and an independent SRS of size$ n\_{2}$ from population 2 with a proportion of successes $p\_{2}$. |
| **Shape** |  |
| **Center** |  |
| **Spread** |  |

Draw the picture of the sampling distribution of $\hat{p}\_{1}-\hat{p}\_{2}$ (Figure 10.2)

*Example: Who Does More Homework?*

**Finding probabilities with the sampling distribution**

Suppose that there are two large high schools, each with more than 2000 students, in a certain town. At School 1, 70% of students did their homework last night. Only 50% of the students at School 2 did their homework last night. The counselor at School 1 takes an SRS of 100 students and records the proportion $\hat{p}\_{1}$that did homework. School 2’s counselor takes an SRS of 200 students and records the proportion $\hat{p}\_{2}$ that did homework. School 1’s counselor and School 2’s counselor meet to discuss the results of their homework surveys. After the meeting, they both report to their principals that $\hat{p}\_{1}-\hat{p}\_{2}$ = 0.10.

**PROBLEM:**

**(a) Describe the shape, center, and spread of the sampling distribution of**$\hat{p}\_{1}-\hat{p}\_{2}$**.**

**Because *n*1*p*1 = 100(0.7) = 70, *n*1 (1 − *p*1) = 100(0.3) = 30, *n*2*p*2 = 200(0.5) = 100, and *n*2 (1 − *p*2) = 200(0.5) = 100 are all at least 10, the sampling distribution of**$\hat{p}\_{1}-\hat{p}\_{2}$ **is approximately Normal. Its mean is *p*1 − *p*2 = 0.70 − 0.50 = 0.20 and its standard deviation is **

**(b) Find the probability of getting a difference in sample proportions**$\hat{p}\_{1}-\hat{p}\_{2}$ **of 0.10 or less from the two surveys. Show your work.**

**Figure 10.3(a)****shows the sampling distribution of**$\hat{p}\_{1}-\hat{p}\_{2}$ **with the desired probability (area) shaded. To find this probability, we use the methods of****Chapter 2****.**



**Figure 10.3a**  (a) Normal curve that approximates the sampling distribution of $\hat{p}\_{1}-\hat{p}\_{2}$ for the homework surveys. We want to find *P*($\hat{p}\_{1}-\hat{p}\_{2}$ ≤ 0.10).

*Standardize.* When $\hat{p}\_{1}-\hat{p}\_{2}$ = 0.10,



*Use*[***Table A***](http://ebooks.bfwpub.com/tps4e/frontmatter/TableA.pdf)*.* The area to the left of *z* = −1.72 under the standard Normal curve is 0.0427. This is the probability we seek. We could use technology to get the desired probability. Using normalcdf(0,0.i,.2,.058) gives 0.0421.

 **Figure 10.3(b)** shows this result.



**Figure 10.3b**  (b) The desired probability can also be expressed as *P*(*z* < −1.72), which represents an area under the standard Normal curve.

**(c) Does the result in part (b) give us reason to doubt the counselors’ reported value? Explain.**

There is only about a 4% chance of getting a difference in sample proportions as small as or smaller than the value of 0.10 reported by the counselors. This does seem suspicious!

**CHECK YOUR UNDERSTANDING**

Your teacher brings two bags of colored goldfish crackers to class. She tells you that Bag 1 has 25% red crackers and Bag 2 has 35% red crackers. Each bag contains more than 500 crackers. Using a paper cup, your teacher takes an SRS of 50 crackers from Bag 1 and a separate SRS of 40 crackers from Bag 2. Let $\hat{p}\_{1}-\hat{p}\_{2}$ be the difference in the sample proportions of red crackers.

**1.** What is the shape of the sampling distribution of $\hat{p}\_{1}-\hat{p}\_{2}$? Why?

*Hint: check sample size condition*

**2.** Find the mean and standard deviation of the sampling distribution. Show your work.

*Hint: use the formulas for mean and standard deviation in your table on the first page of notes.*

**3.** Find the probability that $\hat{p}\_{1}-\hat{p}\_{2}$ is less than or equal to −0.02. Show your work.

*Hint: standardize the difference between two proportions using* $=\frac{statistic-mean}{standard deviation of statistic}$ *, then find the area under the standard normal curve using Table A or normcdf( ) on the calculator.*

**4.** Based on your answer to Question 3, would you be surprised if the difference in the proportion of red crackers in the two samples was $\hat{p}\_{1}-\hat{p}\_{2}$ = −0.02? Explain.

*Hint: what is the probability you found in #3? So small you find it hard to believe the claim?*