Chapter 8: Estimating with Confidence

"Do not put your faith in what statistics say until you have carefully considered what they do not say." William W. Watt

Chapter Overview

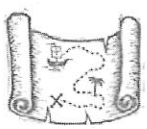
Now that you have learned the basics of probability and sampling distributions, you are ready to begin your formal study of inference. In this chapter, you will use what you have learned about sampling distributions to construct confidence intervals for population proportions and population means. In later chapters, you'll learn how to determine confidence intervals for paired data, differences in means and proportions, and slopes of regression lines. Each of those procedures uses the same approach as the intervals you'll construct in this chapter, so you'll want to get a solid foundation in the basics here!

Sections in this Chapter

Section 8.1: Confidence Intervals: The Basics
Section 8.2: Estimating a Population Proportion
Section 8.3: Estimating a Population Mean

Plan Your Learning

Use the following *suggested* guide to help plan your reading and assignments. Note: your teacher may schedule a different pacing. Be sure to follow his or her instructions!



Read	8.1: pp 467-478	8.1: pp 478-481	8.2: pp 484-495	8.3: pp 499-511
Do	5, 7, 9, 11, 13	17, 19-24, 27, 31, 33	35, 37, 41, 43, 47	49-52, 55, 57, 59, 63

Read	8.3: pp 511-517	Chapter Summary
Do	65, 57, 71, 73, 75-78	Multiple Choice FRAPPY!

Section 8.1: Confidence Intervals: The Basics

Before You Read: Section Summary

In this section you will be introduced to the basic ideas behind constructing and interpreting a confidence interval for a population parameter. You will learn how we can take a point estimate for a population parameter and use what we know about sampling variability to construct an interval of plausible values for the parameter. You will focus on the big ideas in this section. You will want to make sure you understand the different components of a confidence interval as well as the correct interpretation of both the interval and the confidence level. The next two sections will build upon the concepts presented here and focus on the details for estimating proportions and means.

"Where Am I Going?"

Learning Targets:

____ I can interpret a confidence level.
____ I can interpret a confidence interval in context.
____ I can explain that a confidence interval gives a range of plausible values for the parameter.
____ I can explain why each of the three inference conditions – random, Normal, and independent – is important.
____ I can explain how issues like nonresponse, undercoverage, and response bias can influence the interpretation of a confidence interval.
____ I can explain how sample size and level of confidence C affect the margin of error of a confidence interval.

While You Read: Key Vocabulary and Concepts

point estimator:

point estimate:

confidence interval:

margin of error:

confidence level C:

critical value:

conditions for constructing a confidence interval:

After You Read: "Where Am I Now?" Check for Understanding

Concept 1: The Idea of a Confidence Interval

When our goal is to estimate a population parameter, we often must rely on a sample statistic to provide a "point estimate." However, as we learned in the last chapter, that estimate will vary from sample to sample. A confidence interval takes that variation in to account to provide an interval of plausible values, based on the statistic, for the true parameter. All confidence intervals have two main components: an interval based on the estimate that includes a margin of error and a confidence level *C* that reports the success rate of the method used to construct the interval in capturing the parameter in repeated constructions. For example, "C% confident" means C% of all samples of the same size from the population of interest would yield an interval that captures the true parameter. We can then interpret the interval itself to say "We are C% confident that the interval from a to b captures the true value of the population parameter."

Check for Understanding: ____ *I can interpret a confidence level,* ____ *I can interpret a confidence interval in context, and* ____ *I can explain that a confidence interval gives a range of plausible values for the parameter.*

How much do the volumes of bottles of water vary? A random sample of 50 "20 oz." water bottles is collected and the contents are measured. A 90% confidence interval for the population mean μ is 19.10 to 20.74.

a) Interpret the confidence interval in context.

b) Interpret the confidence level in context.

c) Based on this interval, what can you say about the contents of the bottles in the sample? What can you say about the contents of bottles in the population?

Concept 2: Constructing a Confidence Interval

To construct a confidence interval, we must work through three steps. First, you MUST check that the conditions for constructing the interval are met. That is, we must be assured that the data come from a random sample or randomized experiment. The sampling distribution of the statistic must be approximately Normal. And, the individual observations must be independent (which means checking the 10% condition if we're sampling without replacement from a finite population). Second, we construct the interval using the formula

statistic ± (critical value) (standard deviation of the statistic)

where the critical value is determined based on the confidence level C and the standard deviation is based on the sampling distribution of the statistic. Finally, we interpret the interval using the language we learned earlier in this section.

Our goal with confidence intervals is to provide as precise an estimate as possible. That is, we wish to construct a narrow interval that we are confident captures the parameter of interest. We can achieve this in two ways: by decreasing our confidence or by increasing our sample size.

Check for Understanding: ____ *I can explain why each of the three inference conditions – random, Normal, and independent – is important and* ____ *I can explain how issues like nonresponse, undercoverage, and response bias can influence the interpretation of a confidence interval.*

A large company is interested in developing a new bake ware product for consumers. In an effort to determine baking habits of adults, a researcher selects a random sample of 50 addresses in a large, Midwestern, metropolitan area. She calls each selected home in the late-morning to collect information on their baking habits. The proportion of adults who bake at least twice a week is calculated and a 90% confidence interval is constructed.

Discuss whether or not each of the conditions for constructing a confidence interval has been met. If any have not been met, discuss the implications on the interpretation of the interval.

Section 8.2: Estimating a Population Proportion

Before You Read: Section Summary

In the last section, you learned the basic ideas behind confidence intervals. In the next two sections, you will learn how to construct and interpret confidence intervals for proportions and means. You will start by constructing them for proportions, focusing on the application of the four-step process to the procedure.

"Where Am I Going?"

Learning Targets:

____ I can construct and interpret a confidence interval for a population proportion.
____ I can determine critical values for calculating a confidence interval.
____ I can determine the sample size necessary to obtain a level *C* confidence interval for a population proportion with a specified margin of error.

While You Read: Key Vocabulary and Concepts

conditions for estimating *p*:

standard error:

confidence interval for *p*:

sample size for a desired margin of error:

After You Read: "Where Am I Now?"
Check for Understanding

Concept 1: Conditions for Estimating p

When constructing a confidence interval for *p*, it is critical that you begin by checking that the conditions are met. First, check to make sure that the sample was randomly selected or there was random assignment in an experiment. Because the construction of the interval is based on the sampling distribution of $\hat{p}$, next you must ensure that the condition for Normality is met. That is, check to see that $n\hat{p}$ and $n(1-\hat{p})$ are both at least 10. Finally, check for independence of measurements. If there is sampling without replacement, verify that the population of interest is at least 10 times as large as the sample. If all three of these conditions are met, you can safely proceed to construct and interpret a confidence interval for a population proportion *p*.

Concept 2: Constructing a Confidence Interval for p

To construct a confidence interval for a population proportion p, you should follow the four step process introduced in Chapter 1.

- **State** the parameter you want to estimate and at what confidence level.
- **Plan** which confidence interval you will construct and verify that the conditions have been met.
- **Do** the actual construction of the interval using the basic idea from Section 8.1

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

where z^* is the critical value for the standard Normal curve with area C between $-z^*$ and z^*.

- **Conclude** by interpreting the interval in the context of the problem.

Check for Understanding: ____ *I can construct and interpret a confidence interval for a population proportion and* ____ *I can determine critical values for calculating a confidence interval.*

According to a recent study, not everyone can roll their tongue. A researcher observed a random sample of 300 adults and found 68 who could roll their tongue. Use the four-step process to construct and interpret a 90% confidence interval for the true proportion of adults who can roll their tongue.

Concept 3: Choosing the Sample Size

As noted in section 8.1, our goal is to estimate the parameter as precisely as possible. We want high confidence and a low margin of error. To achieve that, we can determine how large a sample size is necessary before proceeding with the data collection. To calculate the sample size necessary to achieve a set margin of error at a confidence level C, we simply solve the following inequality for *n*:

$$z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq ME$$

where $\hat{p}$ is estimated based on a previous study or set to 0.5 to maximize the possible margin of error.

Check for Understanding: ____ *I can determine the sample size necessary to obtain a level C confidence interval for a population proportion with a specified margin of error.*

A researcher would like to estimate the proportion of adults who can roll their tongues. However, unlike the previous example, she'd like the estimate to be within 2% at a 95% confidence level. How large a sample is needed?

Section 8.3: Estimating a Population Mean

Before You Read: Section Summary

In this section, you will continue your study of confidence intervals by learning how to construct and interpret a confidence interval for a mean. While the overall procedure is identical to that for a proportion, there is one major difference. When dealing with means and unknown population standard deviations, we must use a new distribution to determine critical values. You will be introduced to the *t*-distributions and learn how to use them to construct a confidence interval for a population mean.

"Where Am I Going?"

Learning Targets:

____ I can construct and interpret a confidence interval for a population mean.
____ I can determine the sample size required to obtain a level C confidence interval for a population mean with a specified margin of error.
____ I can determine sample statistics from a confidence interval.

While You Read: Key Vocabulary and Concepts

one-sample *z*-interval for a population mean:

t-distribution:

degrees of freedom:

standard error of the sample mean:

one-sample *t*-interval for a population mean:

conditions for inference about a population mean:

robust procedures:

After You Read: "Where Am I Now?" Check for Understanding

Concept 1: Conditions for Estimating μ

Like proportions, when constructing a confidence interval for μ, it is critical that you begin by checking that the conditions are met. First, check to make sure the sample was randomly selected or there was random assignment in an experiment. Because the construction of the interval is based on the sampling distribution of $\bar{x}$, next ensure that the condition for Normality is met. That is, check to see that the population distribution is Normal OR the sample size is at

least 30. Finally, check for independence of measurements. If sampling without replacement was used, verify that the population of interest is at least 10 times as large as the sample. If all three of these conditions are met, you can safely proceed to construct and interpret a confidence interval for a population mean μ.

Concept 2: t-Distributions

When the population standard deviation is unknown, we can no longer model the test statistic with the Normal distribution. Therefore, we can't use critical z^* values to determine the margin of error in our confidence interval. Fortunately, it turns out that when the Normal condition is met, the test statistic calculated using the sample standard deviation s_x has a distribution similar in appearance to the Normal distribution, but with more area in the tails. That is, the statistic

$$t = \frac{\bar{x} - \mu}{s_x / \sqrt{n}}$$

has the t-distribution with (n-1) degrees of freedom. As the sample size and, subsequently, the degrees of freedom increase, the t distribution approaches the standard Normal distribution more and more closely. We calculate standardized t values the same way we calculate z values. However, we must refer to a t table and consider degrees of freedom when determining critical values.

The t-procedures are fairly robust against slight departures from Normality in the population distribution. However, you should exercise caution in using t-procedures when there is evidence of strong skewness or outliers in the sample data.

Check for Understanding:

Use the t table to determine the critical value t^* that you would use for a confidence interval for a population mean μ in the following situations.

a) An 80% confidence interval from a sample with size n = 19

b) A 95% confidence interval from 248 degrees of freedom

c) A 99% confidence interval for a sample with size n = 30

Concept 3: Constructing a Confidence Interval for μ

To construct a confidence interval for a population mean μ when the population standard deviation is unknown, you should follow the four step process introduced in Chapter 1.

- **State** the parameter you want to estimate and at what confidence level.
- **Plan** which confidence interval you will construct and verify that the conditions have been met.
- **Do** the actual construction of the interval using the basic idea from Section 8.1

$$\bar{x} \pm t^* \frac{s_x}{\sqrt{n}}$$

  where t^* is the critical value for the t_{n-1} distribution with area C between $-t^*$ and t^*.
- **Conclude** by interpreting the interval in the context of the problem.

Check for Understanding: ____ *I can construct and interpret a confidence interval for a population mean.*

The amount of sugar in soft drinks is increasingly becoming a concern. To test sugar content, a researcher randomly sampled 8 soft drinks from a particular manufacturer and measured the sugar content in grams/serving. The following data were produced:

26 31 23 22 11 22 14 31

Use these data to construct and interpret a 95% confidence interval for the mean amount of sugar in this manufacturer's soft drinks.

Concept 4: Choosing the Sample Size

Similar to what we did with proportions, to calculate the sample size necessary to achieve a set margin of error for a population mean μ at a confidence level C, we simply solve the following inequality for n:

$$z^* \frac{\sigma}{\sqrt{n}} \leq ME$$

where σ is estimated based on a previous study.

Check for Understanding: ____ *I can determine the sample size required to obtain a level C confidence interval for a population mean with a specified margin of error.*

A researcher would like to estimate the mean amount of time it takes to accomplish a particular task. A previous study indicates the time required varies in the population with a standard deviation of 4 seconds. He would like to estimate the true mean time within 0.5 seconds at 95% confidence. How large a sample is needed?

Chapter Summary: Estimating with Confidence

Statistical inference is the practice of drawing a conclusion about a population based on information gathered from a sample. This chapter introduced us to the practice of estimating a parameter based on a statistic. The underlying logic for confidence intervals is the same whether we are estimating a proportion or a mean. By using what we learned about sampling distributions, we can construct an interval around a point estimate that we are confident captures the parameter of interest. The confidence level itself tells what would happen if we used the construction method for the interval many times for samples of the same size. It is basically the capture rate for all of the constructed intervals. So, when we build a level C interval, we can interpret it by saying "We are C% confident the interval from a to b captures the true parameter of interest."

In the next chapter, we will learn how to test a claim about a parameter. Make sure you continue to practice confidence intervals, though, as they are just as important as the significance tests you are about to learn!

After You Read: "How Can I Close the Gap?"

Complete the vocabulary puzzle, multiple choice questions, and FRAPPY. Check your answers and on your performance on each of the targets.

Target	Got It!	Almost There	Needs Some Work
I can interpret a confidence level in context.			
I can interpret a confidence interval in context.			
I can explain that a confidence interval gives a range of plausible values for the parameter.			
I can explain why each of the three inference conditions – random, Normal, and independent – is important.			
I can explain how issues like nonresponse, undercoverage, and response bias can influence the interpretation of a confidence interval.			
I can construct and interpret a confidence interval for a population proportion.			
I can determine critical values for calculating a confidence interval.			
I can determine the sample size necessary to obtain a level C confidence interval for a population proportion with a specified margin of error.			
I can explain how sample size and level of confidence C affect the margin of error of a confidence interval.			
I can construct and interpret a confidence interval for a population mean.			
I can determine the sample size required to obtain a level C confidence interval for a population mean with a specified margin of error.			
I can determine sample statistics from a confidence interval.			

Did you check "Needs Some Work" for any of the targets? If so, what will you do to address your needs for those targets?

Learning Plan:

Chapter 8 Multiple Choice Practice

Directions. *Identify the choice that best completes the statement or answers the question. Check your answers and note your performance when you are finished.*

1. Gallup Poll interviews 1600 people. Of these, 18% say that they jog regularly. A news report adds: "The poll had a margin of error of plus or minus three percentage points." You can safely conclude that

A. 95% of all Gallup Poll samples like this one give answers within ±3% of the true population value.
B. the percent of the population who jog is certain to be between 15% and 21%.
C. 95% of the population jog between 15% and 21% of the time.
D. we can be 3% confident that the sample result is true.
E. if Gallup took many samples, 95% of them would find that 18% of the people in the sample jog.

2. An agricultural researcher plants 25 plots with a new variety of corn. A 90% confidence interval for the average yield for these plots is found to be 162.72 ± 4.47 bushels per acre. Which of the following is the correct interpretation of the interval?

A. There is a 90% chance the interval from 158.28 to 167.19 captures the true average yield.
B. 90% of sample average yields will be between 158.28 and 167.19 bushels per acre.
C. We are 90% confident the interval from 158.28 to 167.19 captures the true average yield.
D. 90% of the time, the true average yield will fall between 158.28 and 167.19.
E. We are 90% confident the true average yield is 162.72.

3. I collect a random sample of size *n* from a population and from the data collected compute a 95% confidence interval for the mean of the population. Which of the following would produce a wider confidence interval, based on these same data?

A. Use a larger confidence level.
B. Use a smaller confidence level.
C. Use the same confidence level, but compute the interval *n* times. Approximately 5% of these intervals will be larger.
D. Increase the sample size.
E. Nothing can ensure that you will get a larger interval. One can only say the chance of obtaining a larger interval is 0.05.

4. A marketing company discovered the following problems with a recent poll:
I. Some people refused to answer questions
II. People without telephones could not be in the sample
III. Some people never answered the phone in several calls
Which of these sources is included in the ±2% margin of error announced for the poll?

A. Only source I.
B. Only source II.
C. Only source III.
D. All three sources of error.
E. None of these sources of error.

5. You are told that the proportion of those who answered "yes" to a poll about internet use is 0.70, and that the standard error is 0.0459. The sample size

A. is 50.
B. is 99.
C. is 100.
D. is 200.
E. cannot be determined from the information given.

6. The standardized test scores of 16 students have mean $\bar{x}$ = 200 and standard deviation s = 20. What is the standard error of $\bar{x}$?

A.	20
B.	10
C.	5
D.	1.25
E.	0.80

7. A newspaper conducted a statewide survey concerning the 2008 race for state senator. The newspaper took a random sample (assume it is an SRS) of 1200 registered voters and found that 620 would vote for the Republican candidate. Let p represent the proportion of registered voters in the state that would vote for the Republican candidate. A 90% confidence interval for p is

A.	0.517 ± 0.014.
B.	0.517 ± 0.022.
C.	0.517 ± 0.024.
D.	0.517 ± 0.028.
E.	0.517 ± 0.249.

8. After a college's football team once again lost a football game to the college's arch rival, the alumni association decided to conduct a survey to see if alumni were in favor of firing the coach. Let p represent the proportion of all living alumni who favor firing the coach. Which of the following is the smallest sample size needed to guarantee an estimate that's within 0.05 of p at a 95% confidence level?

A.	269
B.	385
C.	538
D.	768
E.	1436

9. An SRS of 100 postal employees found that the average time these employees had worked for the postal service was $\bar{x}$ = 7 years with standard deviation s_x = 2 years. Assume the distribution of the time the population of employees has worked for the postal service is approximately Normal. A 95% confidence interval for the mean time μ the population of postal service employees has spent with the postal service is

A.	7 ± 2.
B.	7 ± 1.984.
C.	7 ± 0.525.
D.	7 ± 0.4.
E.	7 ± 0.2.

10. Do students tend to improve their SAT Mathematics (SAT-M) score the second time they take the test? A random sample of four students who took the test twice earned the following scores.

Student	1	2	3	4
First Score	450	520	720	600
Second Score	440	600	720	630

Assume that the change in SAT-M score (second score − first score) for the population of all students taking the test twice is approximately Normally distributed with mean μ. A 90% confidence interval for μ is

A.	25.0 ± 118.03.
B.	25.0 ± 64.29.
C.	25.0 ± 47.56.
D.	25.0 ± 43.08.
E.	25.0 ± 33.24.

Multiple Choice Answers

Problem	Answer	Concept	Right	Wrong	Simple Mistake?	Need to Study More
1	A	Interpreting Confidence				
2	C	Interpret a Confidence Interval				
3	A	Width of a Confidence Interval				
4	E	Biased Samples				
5	C	Standard Error of $\hat{p}$				
6	C	Standard Error of $\overline{x}$				
7	C	Confidence Interval for p				
8	B	Choosing Sample Size				
9	D	Confidence Interval for μ				
10	C	Confidence Interval for μ (paired data)				

FRAPPY! Free Response AP Problem, Yay!

The following problem is modeled after actual Advanced Placement Statistics free response questions. Your task is to generate a complete, concise response in 15 minutes. After you generate your response, view two example solutions and determine whether you feel they are "complete," "substantial," "developing" or "minimal." If they are not "complete," what would you suggest to the student who wrote them to increase their score? Finally, you will be provided with a rubric. Score your response and note what, if anything, you would do differently to increase your own score.

A machine at a soft-drink bottling factory is calibrated to dispense 12 ounces of cola into cans. A simple random sample of 35 cans is pulled from the line after being filled and the contents are measured. The mean content of the 35 cans is 11.92 ounces with a standard deviation of 0.085 ounce.

a) Construct and interpret a 95% confidence interval to estimate the true mean contents of the cans being filled by this machine.

b) Based on your result from a), does the machine appear to be working properly? Justify your answer.

c) Interpret the confidence level of 95 percent in context.

Student Response 1:

a) One samp-t-int = (11.89, 11.94)

b) There is a 95% chance the true mean of the amount the machine fills cans is captured in this interval.

c) If we took 100 samples, 95 of them would create an interval that captures the true mean.

How would you score this response? Is it substantial? Complete? Developing? Minimal? Is there anything this student could do to earn a better score?

Student Response 2:

a) Conditions: Random sample is given. The cans are independent of each other. Since 35>30, we can assume normality of the sampling distribution.

95% t-interval for the true mean contents:
$11.92 \pm 2.042(0.085/\sqrt{(35)}) = (11.89, 11.94)$

b) We are 95% confident the true mean contents of the cans filled by this machine falls between 11.89 and 11.94 oz. It appears the machine might be underfilling the cans since 12 oz is not in the interval.

c) If we were to take many samples of size 35 and construct intervals from their sample mean contents, 95% of the intervals would capture the true mean contents being dispensed by the filling machine.

How would you score this response? Is it substantial? Complete? Developing? Minimal? Is there anything this student could do to earn a better score?

Scoring Rubric

Use the following rubric to score your response. Each part receives a score of "Essentially Correct," "Partially Correct," or "Incorrect." When you have scored your response, reflect on your understanding of the concepts addressed in this problem. If necessary, note what you would do differently on future questions like this to increase your score.

Intent of the Question

The goal of this question is to determine your ability to construct and interpret a confidence interval and correctly interpret the confidence level in the context of a problem.

Solution

(a) Conditions: Random – The cans were randomly selected.
Independent – There are more than 10(35) cans on the line.
Normal – $n = 35$ (greater than 30), so the sampling distribution of $\bar{x}$ will be approximately normal.

95% CI for μ: $11.92 \pm 2.042(0.085/\sqrt{(35)}) = (11.89, 11.94)$

(b) We are 95% confident that the interval from 11.89 ounces to 11.94 ounces captures the true mean contents of the cans filled by this machine. It appears the machine may be filling less than it is supposed to since 12 is not in the interval.

(c) 95% of intervals constructed from random samples of 35 cans from this machine will be successful in capturing the true mean contents.

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is essentially correct if the response correctly checks the conditions for a one-sample *t* confidence interval for a mean AND correctly calculates the interval. Part (a) is partially correct if the conditions are not properly checked but the interval is correct. Note: the construction of a *z*-interval receives a partial at most.

Part (b) is essentially correct if the response correctly interprets the confidence interval in context AND correctly notes the machine appears to be underfilling because 12 is not contained in the interval. Part (b) is partially correct if the interpretation lacks context OR fails to make a decision about the machine based on the interval.

Part (c) is essentially correct if the response correctly interprets the confidence level in context. Part (c) is partially correct if the interpretation lacks context.

4 **Complete Response**
All three parts essentially correct

3 **Substantial Response**
Two parts essentially correct and one part partially correct

2 **Developing Response**
Two parts essentially correct and no parts partially correct
One part essentially correct and two parts partially correct
Three parts partially correct

1 **Minimal Response**
One part essentially correct and one part partially correct
One part essentially correct and no parts partially correct
No parts essentially correct and two parts partially correct